

insights

and recommendations  
from the

maa

national study of

college

calculus

EDITORS DAVID BRESSOUD VILMA MESA CHRIS RASMUSSEN

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**Insights and Recommendations**  
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**of**  
**College Calculus**

Edited by

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# Chapter 10

## Three Models of Graduate Student Teaching Preparation and Development

Jessica Ellis, *Colorado State University, Fort Collins*

This chapter provides three models for graduate student teaching preparation and development, and example of implementation for each. In this chapter, I use the phrase *graduate student teaching preparation and development (GTPD)* to emphasize that these models go beyond pre-term training in two keys ways: first, they extend into graduate students' roles as recitation leaders or course instructors and second, they prepare graduate students not just for their immediate roles as teaching assistants (TAs), but also for a profession involving teaching. Before describing these three models, I provide an overview of the important roles that graduate students play in undergraduate instruction overall and specifically within the CSPCC study.

Graduate students contribute to undergraduate instruction in two primary ways: as course instructors and as recitation leaders. As course instructors, graduate teaching assistants (GTAs) are in charge of the course just as a lecturer or ladder rank faculty (tenured track or tenured) would be, although they typically lack the experience, education, or time commitment of their faculty counterparts. Graduate students are also frequently employed as recitation leaders, tutors, or graders. Belnap and Allred (2009) found that of the 23 PhD-granting universities surveyed, 35% of the GTAs were the sole instructor for one or two classes, 39% of the GTAs were discussion or recitation leaders, while 25% had other responsibilities such as grading or tutoring<sup>1</sup>. Further, GTAs can be viewed as the next generation of mathematics instructors. Thus, in addition to their immediate contribution to Calculus I teaching, GTAs will contribute significantly to the long-term state of undergraduate mathematics instruction. The teaching preparation and development graduate students receive to prepare them for teaching Calculus I therefore influences both their immediate teaching practices and their long-term pedagogical disposition.

Among the institutions involved in the CSPCC study, graduate students were employed as instructors, as recitation leaders, as tutors in the tutoring centers, and as graders. Of the five doctoral-granting institutions selected for case studies, four employed graduate students in the teaching of Calculus I: two universities mainly employed graduate students as recitation leaders for large Calculus I lectures, while two universities mainly employed graduate students as sole instructors for small sections of Calculus I.<sup>2</sup> Among these four institutions, three distinct GTPD models were used for the teaching preparation and development of graduate students. In this chapter I describe the main features of these three GTPD models, which can be used to inform the creation of a new GTPD program or to help think critically about strengthening an existing program. For each model I answer two questions:

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<sup>1</sup> These percentages do not add up to 100% because institutions could select multiple roles for graduate students.

<sup>2</sup> Graduate students were also employed as graders, tutors, and experienced graduate students taught upper-level courses, such as abstract algebra.

1. What is the guiding philosophy of the model?
2. What are the structural components of the model?

Additionally, I provide one example from the CSPCC case studies to illustrate what each model may look like when embedded within a specific institutional and departmental context. The three models of GTPD are referred to as: *the Apprenticeship model*; *the Coordinated-Innovation model*; and *the Peer-Mentor model*.

## **Apprenticeship Model**

### **What is the guiding philosophy of the model?**

The primary guiding philosophy behind the Apprenticeship model is the desire to transition graduate students into the role of instructor, both as part of their immediate role as GTAs and as their (potential) future role as undergraduate mathematics instructors. Embedded within this philosophy is the belief that people learning a new profession (who will develop a professional identity surrounding it) must participate in the practices of a profession with growing responsibility. This belief is in line with a situated perspective in which learning is viewed as the process of engaging a novice in the practices of a profession with legitimate but peripheral participation (Lave & Wenger, 1991). The term “peripheral” indicates that the practices novices are involved in are less central versions of the authentic practices, or are central practices with limited responsibility. As one clinical psychology professor involved in the Grossman et al. (2009) study said when describing how clinical psychologists are prepared, “if you’re learning to paddle, you wouldn’t practice kayaking down the rapids. You would paddle on a smooth lake to learn your strokes” (p. 2026).

### **What are the structural components of the model?**

The main components of the Apprenticeship model are:

- A three-unit class, inspired by Lesson Study (Lewis, 2004), that takes place during the semester before the graduate student is placed as a course instructor.
- A mentor instructor for whom the mentee acts as a teaching assistant in the class they will be teaching during the semester before the graduate student is placed as a course instructor.
- Weekly course meetings once the graduate student is placed as a course instructor.
- Observations and feedback once the graduate student is placed as a course instructor.

Graduate students are required to participate in a number of teaching development activities, both prior to teaching and while they teach. All new GTAs must attend a one-day seminar led by the mathematics department, with some of this time spent doing practice teaching presentations. During the seminar faculty conduct workshops on topics including pedagogical basics, such as how to write well on the board, as well as more advanced pedagogical topics, such as how to implement cooperative learning. Additionally, all first-year GTAs are assigned a faculty mentor during the orientation session.

In addition to this initial preparation, graduate students must serve as a teaching assistant to a faculty member who mentors them in teaching, and participate in a one-semester course that is based on Japanese Lesson Study (Alvine, Judson, Schein, & Yoshida, 2007).

When serving as teaching assistants to a mentor, graduate students attend the course (typically the same course they will be assigned to teach the following term), help create assignments and assessments, and reflect on class events with the mentor to discuss specific pedagogical decisions. The Lesson Study inspired course involves students in multiple rounds of collaborative lesson development, presenting lessons to one another, giving feedback to one another on these lessons, revising lessons, and delivering the lesson to students. This course is structured to engage graduate students in multiple aspects of being an instructor (preparing and delivering a lesson) with increasing autonomy, while also encouraging graduate students to reflect on their own instruction.

After successful completion of this course and being mentored, graduate students are placed in the classroom as instructors of record. As instructors of record, the graduate students are responsible for most aspects of the course: preparing a course syllabus, preparing for and leading each class, creating assignments, writing quizzes and exams, grading homework and exams, holding office hours, and determining grading scales. Once placed into the classroom

as instructors of record, all GTAs are observed by a faculty member and participate in weekly meetings with other course instructors.

**Example: Public Technical University (PTU).** PTU is a small, public technical institution with approximately 8,000 undergraduates, 20 PhD students, and 15 Master’s students. The majority of these graduate students are supported by teaching assistantships. PTU was selected as a case study institution because it had higher than average positive gains in each student success variable reported (increased interest, enjoyment, and confidence in mathematics). It also had a large number of STEM-intending students and a large number of those students who persisted in their STEM intentions after completing Calculus I. In a typical fall semester, there are approximately 270 Calculus I students in 6 classes of 30-40 students, and typically 5-10 new graduate students. These courses are taught by lecturers, tenured or tenure track faculty, and graduate students, and are coordinated by a long-term lecturer who also serves as the TA trainer and is in charge of placement. The coordination includes uniform assignments and exams. GTAs at PTU serve as course instructors for precalculus, Calculus I, Calculus II, and Introductory Statistics. GTAs may also work in the Mathematica Lab. All calculus students spend one day a week in a computer lab working on Mathematica assignments that are coordinated across sections. This is an uncommon assignment for GTAs and is most often used for GTAs who are not ready to teach their own courses after going through teaching preparation.

The GTPD model was developed by the current department chair when he was the director of the graduate program. When he came into the role of Graduate Director, new graduate students had a 1.5-week orientation in the summer where they would give a few practice lectures and have sessions on how to grade, how to facilitate group work, etc. The department chair said of the program:

*They packed a lot of good information, but it was a week and a half and a lot of them were teaching that first semester. So I participated in that and was just interested, concerned about how well they could really do with that kind of preparation.*

With the support of the chair, the department hired a Director of First-Year Mathematics who developed a course for future graduate student instructors and the mentoring program. Though the person in the role of director of first-year mathematics has changed, the program still relies heavily on the original structure.

## Coordinated Innovation Model

### What is the guiding philosophy of the model?

The primary guiding philosophy behind this model is that Calculus I should be taught in an innovative and student-centered way, in small, highly coordinated classes. This innovation addresses the approach to the content, which is conceptually oriented and application driven, as well as the pedagogical approach, which includes group work and whole class discussions surrounding students’ mathematical activity (rather than the teacher’s). The coordination of these classes ensures that students have similar experiences across different instructors. Further, this coordination helps to support the secondary guiding philosophy: that graduate students can be prepared and supported to successfully implement innovative instruction. This model is also motivated by a third, underlying philosophy: that graduate students can be, as Elaine Seymour termed it (2005), “partners in innovation” and that graduate students who are effectively prepared to implement innovative instruction will likely carry these innovative practices into their future roles as undergraduate mathematics instructors.

### What are the structural components of the model?

The main components of the Coordinated Innovation model are:

- An intensive five-day seminar that occurs the week before graduate students are placed as course instructors.
- Coordination (see Chapter 9).
- Weekly course meetings once the graduate student is placed as an instructor on record.
- Observations and feedback once the graduate student is placed as an instructor on record.

The Coordinated Innovation model prepares and supports GTAs to teach coordinated sections of Calculus I with a conceptually oriented and student centered approach. The main component of this GTPD is a five-day seminar that takes place the week before the semester begins. It provides multiple opportunities for graduate students to present a prepared lesson and get feedback, and a series of presentations aimed to introduce graduate students to the department's approach to calculus, to explain the rationale for the approach, and to share evidence of its success. Many of the materials are reused year after year, with small additions or changes based on facilitators' experiences and feedback from the GTAs. All first-time GTAs participate in the seminar. After the third day of the seminar, the GTA supervisors make course assignments based on availability and graduate student participation in the seminar, specifically their performance in their practice lessons. Most graduate students are placed as course instructors for Calculus I, while some are placed as instructors for precalculus, Calculus II, or are assigned to be tutors in the calculus tutoring center.

Part of the support that graduate students receive once placed in the role of course instructor comes through the coordination of the course. This coordination encompasses common homework, quizzes, exams, and schedule. Because there are many aspects of teaching that are new to graduate students, having these aspects of the course coordinated by an expert instructor allows them to focus their energy on other aspects of instruction. This can be especially helpful when implementing more innovative instruction.

One component of the coordination that is particularly helpful for supporting novice instructors is weekly meetings. These meetings involve all instructors for Calculus I and the course coordinator. These meetings serve as a place to address class management issues including use of group work or how to address specific content, and function not only as an opportunity to cover the logistics of the week, but also as a venue for discussions about student thinking and difficulties.

An experienced graduate student or faculty member observes all GTAs (new and experienced) at least once each term. The observers give feedback to the graduate student. If issues were noted, these are communicated to the GTA, along with concrete ways to address these concerns. In these cases, additional observations are done.

**Example. Large Public University 1 (LPU1).** LPU1 has approximately 1,500 students enrolled in Calculus I each fall, and approximately 125 graduate students. Every Calculus I course has 32 students enrolled, and there are approximately 50 Calculus I sections. GTAs teach the majority of these sections (in fall 2012, 35 out of 50 instructors were first year GTAs). The remaining instructors are experienced GTAs and faculty. LPU1 was selected as a case study institution because the calculus program has been recognized in best practices literature as successfully implementing innovative practices. Additionally, the students and instructors responded at high rates to our survey, and in our analyses of these responses we saw that the Calculus I program at LPU1 did not diminish student persistence, enjoyment, or interest in mathematics (as was the case in our nationwide sample—see Chapter 2). All courses are coordinated by a team of three permanent faculty. All Calculus I courses are taught using an Inquiry Based Learning (IBL) inspired instructional method, which emphasizes student discovery, group work, and conceptual understanding (see [www.inquirybasedlearning.org](http://www.inquirybasedlearning.org) for more information). A typical class consists of a 15 minute lecture, followed by students working on related problems in groups of four, followed by multiple groups presenting their solutions. Each session lasts two hours, so this sequence may be repeated a number of times during each class meeting. The course uses the Hughes-Hallett textbook that was designed to emphasize meaning, applications, and problem solving. Calculus II is structured in a similar way, but Calculus III and IV are different in that they have large lectures of 80 or more students and use a different textbook series.

There is a large mathematics PhD program at LPU1. All graduate students are funded through teaching assistantships unless they obtain research funding. Graduate students at LPU1 act as course instructors for precalculus, Calculus I, or Calculus II. These courses have common midterms, a common final, common online homework, common written homework, and a suggested schedule. As the course instructors, GTAs are responsible for creating quizzes and grading exams. Coordinators develop the schedule, homework, exams, and the final.

## Peer-Mentor Model

### What is the guiding philosophy of the model?

The guiding philosophy for the Peer-Mentor model is that a more experienced GTA is not only capable of preparing and facilitating seminars for GTPD, but that this experience additionally supports interested graduate students in taking a leadership role among the GTAs. Further, that by involving interested and experienced graduate students in the teaching preparation of novice GTAs, this model fosters a community of graduate students around teaching undergraduate mathematics.

### What are the structural components of the model?

The main components of the Peer-Mentor model are:

- An experienced GTA who co-designs and implements the teaching development of GTAs.
- A one-day seminar before the GTAs are placed as recitation leaders.
- An ongoing seminar that occurs periodically throughout the semester.

The Peer-Mentor GTPD model prepares and supports novice GTAs to lead recitation sections of Calculus I. It also prepares more experienced GTAs to be local teaching experts in their current role as Senior TAs and in their (potential) future roles as mathematics faculty. The recitation sessions are designed to go beyond a question and answer session, instead providing opportunities for students to work together on more conceptually oriented problems related to the lecture. The main components of the GTPD model are an initial seminar held before new graduate students begin as recitation leaders and ongoing seminars that address specific issues throughout the year (such as creating and grading exams). The seminars are jointly developed by a senior TA and a supervising faculty member. They are lead by the Senior TA. In addition to developing and running these seminars, the Senior TA observes new GTAs' recitation sections, provides feedback to the GTAs, and, in general, serves as a point of contact for the GTAs regarding issues around the teaching of Calculus I.

All new GTAs attend the seminars, which are optional for more experienced GTAs. The first seminar takes place just before the term begins. Its emphasis is to prepare GTAs for the first day of class. The main goal of the first session is to make sure GTAs are comfortable and confident walking into class on the first day. GTAs participate in required seminars during the first semester, and the Senior TA determines other ongoing teaching preparation. These meetings primarily serve to focus TAs' attention on proctoring and grading exams. These meetings also provide an opportunity for GTAs to investigate student thinking by looking at records of students' solutions and discussing how they would grade them.

The Senior TA is supervised by a faculty member but has the freedom to shape many aspects of the GTPD program, including both the frequency and content of the ongoing seminars. The Senior TA is a key position for the Peer-Mentor model, and fosters a different culture than if these tasks were led solely by a faculty member. Experienced and interested graduate students apply for the position of Senior TA, and the supervising faculty and current Senior TA choose the best candidate. Typically, this is a graduate student who has been successful as a recitation leader for multiple semesters, has leadership qualities, and is likely interested in an academic position involving higher levels of teaching (in addition to research). Once the new Senior TA has been identified, he or she assists the current Senior TA as a way of being trained for this position. Thus, every year there is a Senior TA and a Senior TA in training who work together. The Senior TA has no other TA responsibilities outside of his or her role as Senior TA, but the Senior TA in training has full TA responsibilities in addition to training to be the Senior TA.

**Example: Large Public University 2 (LPU2).** LPU2 is a large public university with approximately 32,000 undergraduates. Due to general requirements, almost all undergraduates take the calculus sequence. LPU2 was selected as a case study institution because it had higher than average positive gains in each student success variable reported (increased interest, enjoyment, and confidence in mathematics), and had a large number of STEM-intending students who persisted in their STEM intentions after completing Calculus I. In a typical fall term, there are about 1,040 students enrolled in mainstream Calculus I intended for STEM majors. Calculus I is taught by visiting faculty and tenured or tenure track faculty. There are typically four sections of fall Calculus I, with between 240 and 320

students. These classes have recitation sections of 40 students led by GTAs and undergraduate teaching assistants. The lecture meets three times a week for 50 minutes and the recitation section meets once a week for 50 minutes. Calculus I is coordinated by a full-time faculty member when it is taught in fall (on sequence), but not when it is taught in winter or spring (off sequence). When it is coordinated, there are uniform assignments and exams across all sections. Tenured faculty can opt out of this coordination (except for the common final) but rarely do so.

At LPU2 both undergraduates and graduate students serve as GTAs for calculus. Their responsibilities are the same, but the Senior TA says that he may “ask a little more of the graduate TA.” These responsibilities include running a recitation section of 40 students, grading a portion of the common exams, holding office hours, and, in some cases, holding a review session before an exam. Graduate students are given two recitation sections, and undergraduates are initially given one. If they receive positive student evaluations, an undergraduate may be given two sections in subsequent terms. Graduate students who have finished their course work and are in advanced candidacy are able to be course instructors for off-sequence calculus courses as well as upper division mathematics courses.

## Conclusion

In this chapter, I provided three models of graduate student teaching preparation and development and examples of institutions that implement each model. The institutions were selected to be a part of the CSPCC case studies because they were determined to be more successful than comparable institutions. After selecting the institutions, we visited them to understand what about them may have contributed to their success. It is no coincidence that among the four PhD institutions that were selected and that employed graduate students in the teaching of Calculus I (as course instructors or recitation leaders), each of the four implemented one of these GTPD models. Certainly these three models are not the only ways to successfully prepare graduate students to become course instructors or recitation leaders; the models instead represent GTPD models that we observed being successfully implemented and that may prove to be effective if adapted at other institutions. However, these GTPD models do not exist as the only factors likely contributing to the success of the selected institutions. Rather, the GTPD models work in conjunction with other successful factors (addressed in the other chapters of this volume).

For instance, each of the GTPD models was implemented at an institution that had a system of coordination (see Chapter 9). This coordination was mandatory for graduate student instructors and recitation leaders, and supported graduate students to be successful as novice instructors in four key ways. First, the coordination served as a control for the type of instruction students would receive in courses or recitation sections taught by graduate students. Novices should not be expected to succeed in all aspects of teaching, and by having certain aspects of the course outside of their control the department can prevent extreme deviations from expected instruction. Second, the coordination allowed graduate students to focus on specific aspects of teaching (such as how to write clearly on the board, how to engage students, what kinds of questions to ask during class, etc.) instead of having to focus on all aspects of teaching (including choosing homework problems, creating exams, determining grading policies, etc.). Third, the coordination was led by someone in the department who was respected for teaching, and whose decisions for the coordinated elements of the course (such as homework problems and exam design) served as a model to novice instructors. Fourth, the coordination fostered a community around the teaching of Calculus I, giving graduate students a network of people to discuss teaching with.

Graduate students at the selected institutions were similarly supported by their departments by being one of many resources for their students (see Chapter 6 for more about student supports), being part of departments that value good and innovative and ambitious teaching (see Chapters 7 and 8), and teaching undergraduate students who are placed in the highest course in which they can succeed (see Chapter 5). The models presented in this chapter should be viewed as a potential ingredient to a larger recipe for Calculus I success, and these ingredients may be adapted to fit the needs and goals of specific departments.

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